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Series approach to the diamagnetism of a disordered granular superconductor network

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Abstract. The exponent ϕ characterizing the divergence of diamagnetic susceptibility of a granular superconductor network in two dimensions is calculated using a low concentration series expansion, and found to be 1.21 ± 0.03 as compared with $\phi = 1.35$ predicted from the scaling relation $\phi = 2\nu - t$ and a recent simulation This discrepancy is probably due to the shortness of our series

Recently, the critical behaviour of the diamagnetic susceptibility of disordered Josephson [1-4] and related arrays [5-6] have been extensively studied. The diamagnetic susceptibility χ is expected to diverge with the exponent ϕ at the percolation threshold in zero external field:

$$\chi \sim (p-p_c)^{-\phi}. \tag{1}$$

The predictions by de Gennes [5] and by Alexander [6] give

$$\phi = 2\nu - t \tag{2}$$

where ν is the correlation length exponent for percolation and t is the conductivity exponent. This relation gives $\phi = 1.36$ if we set $\nu = 1.33$ and $t = 1.30$ in two dimensions. Later, John and Lubensky [3] derived a field theory for a randomly diluted Josephson array and obtained a mean field theory for this model John *et al* [7] have derived (2) using the scaling theory.

Most recently, Roux and Hansen [8] carried out a computer simulation to calculate ϕ using a transfer matrix method. They found $\phi = 1.36 \pm 0.02$ which agrees very well with (2). In this paper, we present a series expansion calculation of χ which gives $\phi = 1.21 \pm 0.03$. This is smaller than the result predicted by (2) and the above simulation result.

The Hamiltonian of the disordered granular Josephson array can be written as

$$H = -\sum_{\langle x,x'\rangle} K_{x,x'} \cos(\theta_x - \theta_{x'} - A_{x,x'}) \tag{3}$$

where $A_{x,x'} = 2\pi/\phi_0 \int_x^{x'} A \cdot dl$ is the line integral of the vector potential A between the nearest-neighbour grains in the units of the flux quantum $\phi_0 = hc/2e$ and $K_{x,x'}$ equals K with probability p and zero with probability $1-p$. The diamagnetic susceptibility χ per site (or per bond) at zero external field can be defined as

$$\chi_0 = \frac{1}{S} \frac{\partial^2 H(\{\theta_i\})}{\partial B^2}\bigg|_{B\to 0} \tag{4}$$

§ Present address Department of Physics, McGill University, Montreal, Quebec, Canada H3A 2T8 where B is the external magnetic field, S is the area of the system, $\{\theta_i\}$ is defined by minimizing the energy H and satisfies the equation

$$\frac{\partial H(\{\theta_i\})}{\partial \theta_i}=0 \qquad i=1,2,\ldots,n_s \tag{5}$$

where n_s is number of sites of the system. To perform series expansion, we define

$$\chi=\sum_G \chi_0(G)P(G)\sim|p_c-p|^{-\phi} \tag{6}$$

where the sum is over all the graphs G and

$$P(G)=p^{n_b(G)}(1-p)^{n_p(G)} \tag{7}$$

is the probability weight for the graph G, where $n_b(G)$ is the number of bonds in G and $n_p(G)$ is the number of perimeter bonds in G.

We calculated χ using a cumulant expansion method [9]. The cumulant of susceptibility χ for a diagram Γ is defined recursively as

$$\chi_c(\Gamma)=\chi(\Gamma)-\sum_{\gamma\in\Gamma}\chi_c(\gamma) \tag{8}$$

where $\chi(\Gamma)$ is the bare value for graph Γ and γ is a subset of Γ. For a graph Γ_1 with dangling bonds, since there is no current in the dangling bond, we have $\chi(\Gamma_1)=\chi(\gamma)$ where γ is the graph which the dangling bond is removed Therefore, the cumulant of $\chi(\Gamma_1)$ is zero. Thus we only need to consider the graph with no free ends. We have generated the diagrams with no free ends and calculated χ up to the order p^{16} on a square lattice. The series coefficients are listed in table 1. We analysed the series using the Padé approximants and the differential Padé approximants [10]. Because we know the percolation threshold p_c on square lattice exactly, we can get ϕ by reading from the pole-residue plot. We obtained $\phi=1.21\pm0.03$ where the error bar comes from fitting the data This value is smaller than the result of Roux amd Hansen. This discrepancy is probably due to the short series we obtained since our series only has 12 non-zero coefficients.

Table 1. The coefficients of the series

n	$c(n)$
1	0.0
2	0.0
3	0.0
4	0.125
5	0.0
6	0.667
7	−0.500
8	4.375
9	−5.467
10	26.188
11	−40.608
12	151.620
13	−272.880
14	920.631
15	−1988.545
16	6318.875

In summary, we have demonstrated that the exponent ϕ of the diamagnetic susceptibility of the granular superconducting network can be calculated using the series expansion method. We obtained $\phi = 1.21 \pm 0.03$ which is smaller than the theoretical prediction and numerical simulation.

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